

Uniformly accelerated mirrors

Part II : Quantum correlations

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Abstract

We study the correlations between the particles emitted by a moving mirror. To this end, we first analyze $\langle T_{\mu\nu}(x)T_{\alpha\beta}(x') \rangle$, the two-point function of the stress tensor of the radiation field. In this we generalize the work undertaken by Carlitz and Willey. To further analyze how the vacuum correlations on \mathcal{J}^- are scattered by the mirror and redistributed among the produced pairs of particles, we use a more powerful approach based on the value of $T_{\mu\nu}$ which is conditional to the detection of a given particle on \mathcal{J}^+ . We apply both methods to the fluxes emitted by a uniformly accelerated mirror. This case is particularly interesting because of its strong interferences which lead to a vanishing flux, and because of its divergences which are due to the infinite blue shift effects associated with the horizons. Using the conditional value of $T_{\mu\nu}$, we reveal the existence of correlations between created particles and their partners in a domain where the mean fluxes and the two-point function vanish. This demonstrates that the scattering by an accelerated mirror leads to a steady conversion of vacuum fluctuations into pairs of quanta. Finally, we study the scattering by two uniformly accelerated mirrors which follow symmetrical trajectories (*i.e.* which possess the same horizons). When using the Davies-Fulling model, the Bogoliubov coefficients encoding pair creation vanish because of perfectly destructive interferences. When using regularized amplitudes, these interferences are inevitably lost thereby giving rise to pair creation.

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Introduction

It is now well understood that the scattering of a quantum radiation field by a non-inertial mirror leads to the production of pairs of particles[1, 2, 3]. However, up to now, most studies have been restricted to the analysis of mean quantities such as the expectation value of the stress tensor $\langle T_{\mu\nu}(x) \rangle$, *i.e.* the one-point function. This analysis is very restrictive in that most of the information concerning the correlations among particles is ignored. In particular, $\langle T_{\mu\nu}(x) \rangle$ cannot be used to identify the relationships between the particles and their partners.

In this paper, it is our intention to go beyond the mean field approach. To this end, we first study the (connected part of the) two-point function $\langle T_{\mu\nu}(x)T_{\alpha\beta}(x') \rangle_c$. In this, we complete the analysis undertaken by Carlitz and Willey [4] and Wilczek [5], see also [6, 7]. Our motivations are the following. First, since $T_{\mu\nu}$ is the source of gravity, if one wishes to go beyond the semi-classical treatment, *i.e.* Einstein equations driven by the mean $\langle T_{\mu\nu} \rangle$, it is imperative to gain some experience concerning the two-point function since it governs the metric fluctuations about the mean background geometry [8, 9, 10, 11]. Secondly, we wish to relate the analysis of $\langle T_{\mu\nu}(x)T_{\alpha\beta}(x') \rangle_c$ to an alternative approach[12, 13, 14, 3] of correlations which was used to reveal the space time distribution of the correlations among charged pairs produced in a constant electric field and among Hawking quanta emerging from a black hole. This method is based on the value of $T_{\mu\nu}$ which is conditional to the detection of a specific quantum (or specific quanta) on \mathcal{J}^+ . We shall show that the two approaches are closely related and that the second one is more powerful to identify the correlations between the particles and their partners. Finally, the quantum correlations within the fluxes emitted by a mirror constitute an interesting subject *per se*.

In this respect, it is particularly interesting to study the correlations in the fluxes emitted by a uniformly accelerated mirror. Indeed, these fluxes possess, on one hand, strong interferences which lead to a vanishing mean flux and, on the other hand, very high frequencies associated with the diverging blue shift effects encountered when the mirror enters or leaves space-time. In order to tame this singular behavior, one needs to abandon the original Davies-Fulling model [1] and use a self-interacting model described by an action [15, 16, 17]. In this paper, we shall compare the two-point functions computed with the Davies-Fulling model and this self-interacting model.

Because of the strong interferences in the case of uniform acceleration, we shall see that the analysis of the two-point function is not sufficient to properly isolate the correlations among the produced particles. To complete the analysis, we therefore use the conditional value of $T_{\mu\nu}$. By an appropriate choice of the detected quantum on \mathcal{J}^+ , we unravel correlations among the two members in a produced pair even in domains where the mean flux and the two-point function vanish. These correlations show that the scattering by a uniformly accelerated mirror leads to a steady conversion of vacuum fluctuations into pairs of particles, something which could not be seen from the expressions of the mean flux and the two-point function which both vanish. Another nice property of this alternative approach is that the wave packet of the detected particle can be chosen in such a way that the former regularization of the scattering amplitudes is no longer necessary. We hope that this double and complementary analysis of observables in the presence of very high frequencies can lead to a better understanding of the “trans-Planckian” physics, *i.e.* the fact that Hawking radiation[18, 19, 20, 21, 14, 11], and cosmological density fluctuations[22, 23] arise from ultra-high energy configurations.

Finally, to illustrate the necessity of using regular scattering amplitudes, we study the scattering by two uniformly accelerated mirrors which follow symmetrical trajectories (*i.e.* which possess the same horizons). When using the Davies-Fulling model, the Bogoliubov coefficients governing pair creation identically vanish. This vanishing follows from perfectly destructive interferences between the two mirrors, a phenomenon related to what Gerlach[24] called a perfect interferometer and which was also found when considering the fluxes emitted by two accelerating black holes[25, 26]. When using regulated amplitudes, we show that these interferences are inevitably lost and that the total energy emitted is the sum of the energy emitted by each mirror. It thus appears that the perfect interferences are an artifact due to the oversimplification of the description of the scattering. A similar conclusion can be reached when taking into account recoil effects [15, 27]. This further legitimizes the use of regulated scattering amplitudes.

We have organized the paper as follows. In Sec. 1, we recall the basic properties of the self-interacting model. Sec. 2 is devoted to the study of the two-point correlation function. In Sec. 3 we compute the conditional value of $T_{\mu\nu}$ and in Sec. 4 we study the scattering by two uniformly accelerated mirrors.

1 The Lagrangian model

In [17], our aim was to obtain regular expressions for the fluxes and the energy emitted by a uniformly accelerated mirror. To this end, the scattering of the scalar field Φ by the mirror was described by a self-interacting model based on an action.

The action density is localized on the mirror trajectory $x_{cl}^\mu(\tau)$ where τ is the proper time. To preserve the linearity of the scattering, the density is a quadratic form of the field Φ . Since the field is massless, IR divergences appear in the transition amplitudes. To get rid of these problems, it is sufficient to use a density which contains two time derivatives. The interacting Lagrangian we shall use is

$$\begin{aligned} L_{int} &= -g_0 \int d\tau g(\tau) \int d^2x \delta^2(x^\mu - x_{cl}^\mu(\tau)) \frac{dx_{cl}^\mu}{d\tau} \frac{dx_{cl}^\nu}{d\tau} (\partial_\mu \Phi^\dagger \partial_\nu \Phi + \partial_\mu \Phi \partial_\nu \Phi^\dagger) \\ &= -g_0 \int d\tau g(\tau) (\partial_\tau \Phi^\dagger \partial_\tau \Phi + \partial_\tau \Phi \partial_\tau \Phi^\dagger) . \end{aligned} \quad (1)$$

Here, g_0 is the coupling constant. The real function $g(\tau)$ controls the time dependence of the interaction. When the interaction lasts $2T$, its normalization is given by $\int d\tau g(\tau) = 2T$. The two terms in the parentheses imply that L_{int} is charge-less. Hence the transition amplitudes will be invariant under charge conjugation.

We work in the interacting picture. Therefore, the charged field evolves freely, *i.e.* according to the d'Alembert equation,

$$\square \Phi(t, z) = 4\partial_U \partial_V \Phi(U, V) = 0 . \quad (2)$$

Since the field is massless, it is useful to use the light-like coordinates $U, V = t \mp z$. The free field Φ can be decomposed as

$$\Phi(U, V) = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} \left(a_\omega^U e^{-i\omega U} + a_\omega^V e^{-i\omega V} + b_\omega^{U\dagger} e^{i\omega U} + b_\omega^{V\dagger} e^{i\omega V} \right) . \quad (3)$$

The annihilation and creation operators of left and right-moving particles (and antiparticles) are constant and obey the usual commutation relations

$$[a_\omega^i, a_{\omega'}^{j\dagger}] = \delta^{ij} \delta(\omega - \omega') \quad , \quad [b_\omega^i, b_{\omega'}^{j\dagger}] = \delta^{ij} \delta(\omega - \omega') \quad , \quad (4)$$

where the indices i, j stand for U and V . All other commutators vanish. In the interacting picture, the states evolve through the action of the time-ordered operator $T e^{iL_{int}}$. When the initial state is vacuum, the state on \mathcal{J}^+ is given, up to second order in g_0 , by

$$T e^{iL_{int}} |0\rangle = |0\rangle + iL_{int}|0\rangle + \frac{(iL_{int})^2}{2}|0\rangle + |D\rangle . \quad (5)$$

The ket $|D\rangle$ contains terms arising from time-ordering. These terms do not contribute to the total energy emitted (see [16]). Hence we drop $|D\rangle$ from now on.

When working in the vacuum, to obtain the mean flux $\langle T_{\mu\nu} \rangle$ and the two-point function $\langle T_{\mu\nu} T_{\alpha\beta} \rangle$ to order g_0^2 , it is sufficient to develop the scattering amplitudes to first order in g_0 . To this order, the amplitude describing the scattering of an incoming quantum of frequency ω' to an outgoing of frequency ω is given by

$$A_{\omega\omega'}^{ij*} \equiv \langle 0 | a_\omega^i (1 + iL_{int}) a_{\omega'}^{j\dagger} | 0 \rangle_c , \quad (6)$$

where the subscript $\langle \rangle_c$ means that only the connected graphs are kept. Similarly, the spontaneous pair production amplitude reads

$$B_{\omega\omega'}^{ij} \equiv \langle 0 | a_\omega^i b_{\omega'}^j iL_{int} | 0 \rangle . \quad (7)$$

Both non-local and local objects are easily obtained in terms A and B . For instance, to order g_0^2 , the mean number of spontaneously created left-moving particles of frequency ω is given by

$$\langle N_\omega^V \rangle \equiv \langle 0 | L_{int} a_\omega^{V\dagger} a_\omega^V L_{int} | 0 \rangle = \int_0^\infty d\omega' \left(|B_{\omega\omega'}^{VV}|^2 + |B_{\omega\omega'}^{VU}|^2 \right) . \quad (8)$$

Then, the (subtracted) integrated energy is, as usual,

$$\langle H^V \rangle = 2 \int_0^\infty d\omega \, \omega \langle N_\omega^V \rangle . \quad (9)$$

The factor of 2 stands for particles + antiparticles, which contribute equally. One can also compute the local flux of energy. The corresponding operator is $T_{VV} = \partial_V \Phi^\dagger \partial_V \Phi + \partial_V \Phi \partial_V \Phi^\dagger$. Its vacuum expectation value is given by

$$\begin{aligned} \langle T_{VV}(V) \rangle &\equiv \langle 0 | e^{-iL_{int}} T_{VV} e^{iL_{int}} | 0 \rangle_c - \langle 0 | T_{VV} | 0 \rangle \\ &= 2 \sum_{j=U,V} \int \int_0^\infty d\omega d\omega' \frac{\sqrt{\omega\omega'}}{2\pi} e^{-i(\omega' - \omega)V} \left(\int_0^\infty dk \, B_{\omega k}^{Vj*} B_{\omega' k}^{Vj} \right) \\ &\quad - 2 \text{Re} \left\{ \sum_{j=U,V} \int \int_0^\infty d\omega d\omega' \frac{\sqrt{\omega\omega'}}{2\pi} e^{-i(\omega' + \omega)V} \left(\int_0^\infty dk \, A_{\omega k}^{Vj*} B_{\omega' k}^{Vj} \right) \right\} . \end{aligned} \quad (10)$$

We have subtracted the average value of T_{VV} in the vacuum in order to remove the zero point energy. When integrated over all V , the first term of Eq.(10) determines the (positive) energy $\langle H^V \rangle$ of Eq.(9). The second term clearly integrates to 0.

The above model can easily be related to the original Davies-Fulling one [1, 2], where the field obeys

$$\square\Phi(U, V) = 0 \quad \text{and} \quad \Phi(V, V_{cl}(U)) = 0, \quad (11)$$

where $V_{cl}(U)$ is the mirror trajectory expressed in null coordinates. On the right of the trajectory, the Bogoliubov coefficients are given by the overlaps between initial V modes of frequency ω defined on \mathcal{J}_R^- and *out*-modes of frequency ω' defined on \mathcal{J}_R^+ :

$$\alpha_{\omega\omega'}^* \equiv (\varphi_\omega^{U,out}, \varphi_{\omega'}^{V,in}) = - \int_{-\infty}^{+\infty} dU \frac{e^{i\omega U}}{\sqrt{4\pi\omega}} i \partial_U \frac{e^{-i\omega' V_{cl}(U)}}{\sqrt{4\pi\omega'}}, \quad (12)$$

$$\beta_{\omega\omega'}^* \equiv (\varphi_\omega^{U,out*}, \varphi_{\omega'}^{V,in}) = - \int_{-\infty}^{+\infty} dU \frac{e^{-i\omega U}}{\sqrt{4\pi\omega}} i \partial_U \frac{e^{-i\omega' V_{cl}(U)}}{\sqrt{4\pi\omega'}}. \quad (13)$$

Two properties are worth remembering. In the Davies-Fulling model, when starting from vacuum, the flux of energy emitted by the mirror is given by Eq.(10) with $A_{\omega\omega'}^{UU}$ and $B_{\omega\omega'}^{UU}$ respectively replaced by $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$, and $A_{\omega\omega'}^{VV}$, $B_{\omega\omega'}^{VV}$ sent to zero. Secondly, $A_{\omega\omega'}^{UV} = ig_0\alpha_{\omega\omega'}$ and $B_{\omega\omega'}^{UV} = -ig_0\beta_{\omega\omega'}$ for all trajectories $V = V_{cl}(U)$ when using $\Phi^\dagger i \partial_\tau \Phi$ instead of $\partial_\tau \Phi^\dagger \partial_\tau \Phi + \partial_\tau \Phi \partial_\tau \Phi^\dagger$ in Eq.(1) and when putting $g(\tau) = 1$, see [16] for more details.

These two properties guarantee that $\langle T_{VV}(V) \rangle$ behave similarly whether one uses the Lagrangian or the Davies-Fulling model to describe the scattering. It is only for trajectories which lead to singular fluxes when using the Davies-Fulling model that the two descriptions can significantly differ because the coupling function $g(\tau)$ in Eq.(1) can be chosen so as to obtain regular expressions.

2 The two-point correlation function

To analyze the quantum correlations among the particles emitted by the mirror, we first study the two-point function of $T_{\mu\nu}$. Given the Lagrangian defined in Eq.(1), when a U quantum is detected on \mathcal{J}_R^+ (the right hand side of \mathcal{J}^+ , see Fig.1), its partner can be either a U or a V quantum, emitted respectively toward \mathcal{J}_R^+ or \mathcal{J}_L^+ .

In this Section, for reasons of simplicity, we mainly focus on U/V correlations and study the two-point function $\langle T_{UU} T_{VV} \rangle_c$. Indeed, in the absence of the mirror, these correlations vanish. Hence, if $\langle T_{UU} T_{VV} \rangle_c \neq 0$, it results from the scattering and not from pre-existing correlations which exist in the vacuum, see Sec. 2.1. This is not the case for $\langle T_{VV} T_{VV} \rangle_c$ which originates both from the scattering as well as from pre-existing correlations. Moreover, since these two channels interfere, the expressions are much more complicated.

2.1 Initial correlations on \mathcal{J}^- , before the scattering

On \mathcal{J}_R^- , when the trajectory does not enter space through it, the field is unscattered. Therefore, when working in the vacuum, the two-point function is given by

$$\begin{aligned} C_{vac}(V, V') &\equiv \langle 0 | T_{VV}(U = -\infty, V) T_{VV}(U' = -\infty, V') | 0 \rangle_c \\ &= (2\partial_V \partial_{V'} W_{vac}(V, V'))^2 = \frac{1}{4\pi^2} \frac{1}{(V - V' - i\epsilon)^4}, \end{aligned} \quad (14)$$

where the subscript c means that only connected graphs are kept and where $W_{vac}(V, V')$ is the V -part of the vacuum Wightman function

$$W_{vac}(V, V') \equiv \langle 0 | \Phi(U = -\infty, V) \Phi^\dagger(U' = -\infty, V') | 0 \rangle = -\frac{1}{4\pi} \ln(V - V' - i\epsilon) . \quad (15)$$

Eq.(14) is valid for the Davies-Fulling model and for the Lagrangian model. It applies both for an inertial and for a uniformly accelerated mirror in L , since the past null infinity \mathcal{J}_R^- lies on the past of the mirror, see Fig.2. Had the mirror entered space through \mathcal{J}_R^- , a prescription should have been adopted to define the in-vacuum on \mathcal{J}_R^- in the presence of a mirror, see Sec. 1.2 in [17].

2.2 Correlations between \mathcal{J}^- and \mathcal{J}^+

When the field is initially in vacuum, the correlations between \mathcal{J}^- and \mathcal{J}^+ are governed by the (connected) two-point function

$$C^{+/-}(U, V') \equiv \langle 0 | T_{UU}(U, V = +\infty) T_{VV'}(U' = -\infty, V') | 0 \rangle_c . \quad (16)$$

In this expression, written in the Heisenberg picture, only T_{UU} is evaluated on \mathcal{J}^+ . Hence, in the interacting picture, $C^{+/-}(U, V')$ is given by

$$C^{+/-}(U, V') \equiv \langle 0 | e^{-iL_{int}} T_{UU}(U) e^{iL_{int}} T_{VV'}(V') | 0 \rangle_c , \quad (17)$$

where T_{UU} and $T_{VV'}$ are now expressed in terms of the free field of Eq.(3). To second order in the coupling constant and when neglecting again the $|D\rangle$ term of Eq.(5), we get

$$\begin{aligned} C^{+/-}(U, V') &= \langle 0 | L_{int} T_{UU} L_{int} T_{VV'} | 0 \rangle_c \\ &\quad - \frac{1}{2} \langle 0 | (L_{int} L_{int} T_{UU} + T_{UU} L_{int} L_{int}) T_{VV'} | 0 \rangle_c . \end{aligned} \quad (18)$$

To compute this expression, it is convenient to introduce the functions

$$F(U, V') \equiv i \langle 0 | \partial_U \Phi(U) \partial_{V'} \Phi^\dagger(V') L_{int} | 0 \rangle \quad (19)$$

$$= - \int \int_0^\infty d\omega d\omega' \frac{\sqrt{\omega\omega'}}{4\pi} B_{\omega\omega'}^{UV} e^{-i(\omega U + \omega' V')} \quad (20)$$

and

$$G(U, V') \equiv i \langle 0 | \partial_U \Phi(U) L_{int} \partial_{V'} \Phi^\dagger(V') | 0 \rangle \quad (21)$$

$$= \int \int_0^\infty d\omega d\omega' \frac{\sqrt{\omega\omega'}}{4\pi} A_{\omega\omega'}^{UV*} e^{-i(\omega U - \omega' V')} . \quad (22)$$

From these equations, we see that F is expressed in terms of the pair production amplitude B whereas G is a function of the scattering amplitude A . Using Eqs.(19) and (21), one can rewrite the correlation function in the following form

$$C^{+/-}(U, V') = (F^*(U, V') + G(U, V'))^2 . \quad (23)$$

Before applying Eq.(23) to inertial and accelerated trajectories, it is interesting to compute Eq.(16) in the Davies-Fulling model. In this case, one gets

$$\begin{aligned} C_{DF}^{+/-}(U, V') &= 2 \left(\partial_U \partial_{V'} W_{DF}^{+/-}(U, V') \right)^2 \\ &= \frac{1}{4\pi^2} \frac{(dV_{cl}/dU)^2}{(V_{cl}(U) - V' - i\epsilon)^4} , \end{aligned} \quad (24)$$

where the relevant part of the scattered Wightman function is given by

$$\begin{aligned} W_{DF}^{+/-}(U, V') &= \langle 0 | \Phi(U, V = +\infty) \Phi^\dagger(U' = -\infty, V') | 0 \rangle \\ &= \frac{1}{4\pi} \ln(V_{cl}(U) - V' - i\epsilon) . \end{aligned} \quad (25)$$

Eqs.(23) and (24) show that the two-point function is not real. In this regard, it is worth making the following remarks. First we note that $C_{DF}^{+/-}$ diverges only when $V' \rightarrow V_{cl}(U)$, *i.e.* at the classical image point. Similarly, we shall see that $C^{+/-}$ of Eq.(23) also diverges in this limit but is otherwise finite. The fact that no regularization is needed to evaluate it follows from the fact that only connected graphs have been kept in Eq.(17).

Secondly, we note that the imaginary character of $C_{DF}^{+/-}$ arises only from its singular limit. The sign of ϵ encodes the fact that only positive frequencies enter in the Wightman function Eq.(15). Therefore, on one hand, the limit $\epsilon \rightarrow 0$ can be taken when evaluating the two-point functions when $V' \neq V_{cl}(U)$. On the other hand however, the $i\epsilon$ prescription must be kept when using $C^{+/-}$ to obtain the correlations of integrated operators, *e.g.* $\langle H^U T_{VV} \rangle$ where $H^U = \int dU T_{UU}$. Indeed, the definition of the integral over U requires that the limit $\epsilon \rightarrow 0$ be taken after having performed the integral, see [28] where it is shown that the same procedure should be used to properly evaluate the energy in the Rindler vacuum. In this sense, the two-point functions should be viewed as distributions.

2.2.1 Inertial mirror

In the case of an inertial mirror, the canonical³ trajectory reads $V_{cl}(U) = U$. The corresponding space-time diagram is pictured in Fig.1. In the Davies-Fulling model, using Eq.(24), the two-point function reads

$$C_{DF,inert}^{+/-}(U, V') = \frac{1}{4\pi^2} \frac{1}{(U - V' - i\epsilon)^4} . \quad (26)$$

The small distance divergence comes from the vacuum configurations emerging from \mathcal{J}_R^- at V' and which have been reflected on the mirror at $V' = V_{cl}(U) = U$. We therefore recover the divergence which existed in the vacuum on \mathcal{J}^- , see Eq.(14).

To compute the corresponding two-point function in the Lagrangian model, we use Eq.(23). Eqs.(19) and (21) applied to the inertial trajectory give

$$\begin{aligned} F_{inert}(U, V') &= -2ig_0 \int_{-\infty}^{+\infty} dt g(t) \partial_U \partial_t W_{vac}(U, t) \partial_{V'} \partial_t W_{vac}(V', t) \\ &= \frac{-2ig_0}{(4\pi)^2} \int_{-\infty}^{+\infty} dt g(t) \frac{1}{(U - t - i\epsilon)^2} \frac{1}{(V' - t - i\epsilon)^2} \end{aligned} \quad (27)$$

³Generally, inertial trajectories read $V_{cl}(U) = |\xi|(U - U_0) + V_0$. They all provide the same two-point functions by applying $U' = \sqrt{|\xi|}(U - U_0)$, $V' = (V - V_0)/\sqrt{|\xi|}$.

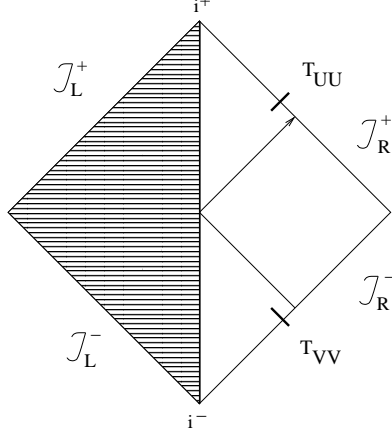


Figure 1: The Penrose diagram of space-time around an inertial mirror. Being interested in the correlations between \mathcal{J}_R^- and \mathcal{J}_R^+ , the dashed region is of no interest.

$$\begin{aligned}
G_{inert}(U, V') &= -2ig_0 \int_{-\infty}^{+\infty} dt g(t) \partial_U \partial_t W_{vac}(U, t) \partial_t \partial_{V'} W_{vac}(t, V') \\
&= \frac{-2ig_0}{(4\pi)^2} \int_{-\infty}^{+\infty} dt g(t) \frac{1}{(U-t-i\epsilon)^2} \frac{1}{(t-V'-i\epsilon)^2} .
\end{aligned} \tag{28}$$

To obtain local expressions of F_{inert} and G_{inert} , we express $g(t)$ with its Fourier components and perform the integrations over t by the method of residues. Since Minkowski vacuum contains only positive frequency, it is appropriate to decompose $g(t)$ as $g(t) = g_+(t) + g_-(t)$ where g_+ contains only positive frequencies:

$$g_+(t) = \int_0^\infty d\omega g_\omega e^{-i\omega t} \quad \text{with} \quad g_\omega \equiv \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt g(t) e^{i\omega t} . \tag{29}$$

Then we get

$$F_{inert}(U, V') = -\frac{g_0}{4\pi} \left[\partial_U \left(\frac{g_+(U)}{(U-V'-i\epsilon)^2} \right) + \partial_{V'} \left(\frac{g_+(V')}{(V'-U-i\epsilon)^2} \right) \right] \tag{30}$$

$$G_{inert}(U, V') = -\frac{g_0}{4\pi} \left[\partial_U \left(\frac{g_+(U)}{(U-V'-i\epsilon)^2} \right) - \partial_{V'} \left(\frac{g_-(V')}{(V'-U+i\epsilon)^2} \right) \right] . \tag{31}$$

To obtain the value of $C_{inert}^{+/-}(U, V')$ off the image point $U = U_{cl}(V') = V'$, we can take the limit $\epsilon \rightarrow 0$. Using Eq.(23), we get

$$C_{inert}^{+/-}(U, V')|_{\epsilon=0} = \frac{g_0^2}{(4\pi)^2} \left[\partial_U \left(\frac{g(U)}{(U-V')^2} \right) \right]^2 . \tag{32}$$

Two remarks should be made. On one hand, when $g(t) \equiv 1$, one recovers Eq.(26) up to two differences. The first one is the g_0^2 pre-factor due to the perturbative expansion. The second one is the power of the pole (of order 6 instead of 4). This discrepancy arises from the fact that we have taken a Lagrangian with two time derivatives (see Eq.(1)). Had we taken the current $J = \Phi^\dagger i \overleftrightarrow{\partial}_\tau \Phi$ instead of $\partial_\tau \Phi^\dagger \partial_\tau \Phi + \partial_\tau \Phi \partial_\tau \Phi^\dagger$, we would have obtained a fourth order pole in Eq.(32). However, in that case, the expressions in Fourier transform, *i.e.* Eqs.(20) and (22), would have been divergent in the low-frequency limit.

On the other hand, when $g(t)$ possesses a compact support, the correlator identically vanishes, as it should do by causality, when the *outgoing* configurations (here the U configurations) lie outside the support of $g(t)$, *i.e.* when they cross the trajectory without being scattered by the mirror. Hence, unlike in the Davies-Fulling model, $C_{inert}^{+/-}(U, V')$ is not symmetric in U, V' .

2.2.2 Classical scattering by a uniformly accelerated mirror

We first look at classically reflected configurations, *i.e.* at configurations which have support for $V' < 0$ and $U > 0$ when the uniformly accelerated trajectory follows $V_d(U) = -1/a^2 U$ in the left Rindler wedge, see Fig.2. This situation corresponds to what we just analyzed for an inertial trajectory.

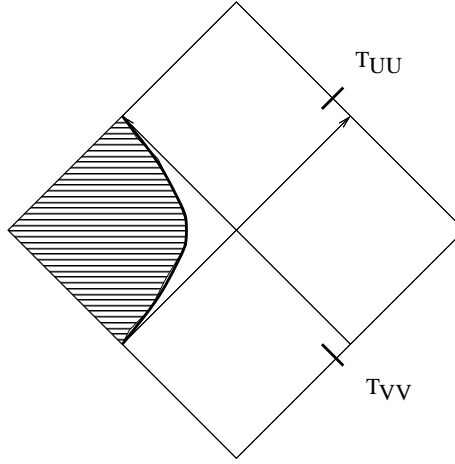


Figure 2: The Penrose diagram of space-time around a uniformly accelerated mirror in the left Rindler wedge. As before, we consider only configurations on the right of the trajectory.

In the Davies-Fulling model, the relevant part of the Wightman function, Eq.(25), is given by

$$W_{DF,unif.acc.}^{+/-}(U, V') = \frac{1}{4\pi} \Theta(U) \ln \left(-1/(a^2 U) - V' - i\epsilon \right) . \quad (33)$$

Then, Eq.(24) gives

$$C_{DF,unif.acc.}^{+/-}(U, V') = \frac{1}{4\pi^2} \Theta(U) \frac{(1/a^2 U^2)^2}{(-1/(a^2 U) - V' - i\epsilon)^4} , \quad (34)$$

up to an ill-defined singular contribution on $U = 0$. Using the Rindler coordinates

$$v'_L = -\frac{1}{a} \ln(-aV') \quad \text{and} \quad u_L = \frac{1}{a} \ln(aU) , \quad (35)$$

the “Rindler” correlation function is

$$\begin{aligned} C_{DF,unif.acc.}^{+/-}(u_L, v'_L) &\equiv \langle T_{uu}(u_L) T_{vv}(v'_L) \rangle \\ &= \left(\frac{dU}{du_L} \right)^2 \left(\frac{dV'}{dv'_L} \right)^2 C_{DF,unif.acc.}^{+/-}(U, V') \\ &= \left[\frac{a^2}{8\pi \sinh^2 \left(\frac{a(u_L - v'_L - i\epsilon)}{2} \right)} \right]^2 . \end{aligned} \quad (36)$$

When comparing this expression⁴ to that of an inertial trajectory, Eq.(26), one finds the following correspondence: Eq.(36) is exactly the expression one would have obtained for $C^{+/-}$ in the case of an inertial mirror in a thermal bath with $T = a/2\pi$, see [5]. This is no surprise since the scattering of Rindler modes by an accelerated mirror is identical (in fact trivial when using the Davies-Fulling model) to the scattering of Minkowski modes by an inertial mirror, and since the two-point function W_{vac} of Eq.(15) is thermal when expressed in terms of Rindler coordinates. We shall now verify that the same correspondence applies to the Lagrangian model.

When applying this model to the uniformly accelerated trajectory, Eqs.(19) and (21) give

$$F_{unif.acc.}(U, V') = \frac{-2ig_0a^4}{(16\pi)^2} e^{a(v'_L - u_L)} \int_{-\infty}^{+\infty} d\tau g(\tau) \frac{1}{\sinh^2(\frac{a(u_L - \tau - i\epsilon)}{2})} \frac{1}{\sinh^2(\frac{a(v'_L - \tau - i\epsilon)}{2})} \quad (38)$$

$$G_{unif.acc.}(U, V') = \frac{-2ig_0a^4}{(16\pi)^2} e^{a(v'_L - u_L)} \int_{-\infty}^{+\infty} d\tau g(\tau) \frac{1}{\sinh^2(\frac{a(u_L - \tau - i\epsilon)}{2})} \frac{1}{\sinh^2(\frac{a(\tau - v'_L - i\epsilon)}{2})} \quad (39)$$

As in the inertial case, one can perform the integrals when using the following decomposition

$$\frac{1}{\sinh^2(x)} = \sum_{n=-\infty}^{+\infty} \frac{1}{(x - in\pi)^2}, \quad (40)$$

and by introducing the function

$$\bar{g}_+(\tau) = \int_{-\infty}^{+\infty} d\lambda \frac{g\lambda e^{-i\lambda\tau}}{1 - e^{-2\pi\lambda/a}} \quad \text{with} \quad g_\lambda \equiv \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\tau g(\tau) e^{i\lambda\tau}, \quad (41)$$

and $\bar{g}_-(\tau) = \bar{g}_+^*(\tau)$. We get

$$F_{unif.acc.}(U, V') = -\frac{g_0}{4\pi} \frac{a^2}{4} e^{a(v'_L - u_L)} \left[\partial_{u_L} \left(\frac{\bar{g}_+(u_L)}{\sinh^2(\frac{a}{2}(u_L - v'_L - i\epsilon))} \right) + \partial_{v'_L} \left(\frac{\bar{g}_+(v'_L)}{\sinh^2(\frac{a}{2}(v'_L - u_L - i\epsilon))} \right) \right] \quad (42)$$

$$G_{unif.acc.}(U, V') = -\frac{g_0}{4\pi} \frac{a^2}{4} e^{a(v'_L - u_L)} \left[\partial_{u_L} \left(\frac{\bar{g}_+(u_L)}{\sinh^2(\frac{a}{2}(u_L - v'_L - i\epsilon))} \right) - \partial_{v'_L} \left(\frac{\bar{g}_-(v'_L)}{\sinh^2(\frac{a}{2}(v'_L - u_L + i\epsilon))} \right) \right]. \quad (43)$$

Since $\bar{g}_+(\tau) + \bar{g}_-(\tau) = g(\tau)$, when taking the limit $\epsilon \rightarrow 0$, the correlation function reads

$$C_{unif.acc.}^{+/-}(u_L, v'_L) = \frac{g_0^2}{(4\pi)^2} \left[\frac{a^2}{4} \partial_{u_L} \left(\frac{g(u_L)}{\sinh^2(\frac{a}{2}(u_L - v'_L))} \right) \right]^2, \quad (44)$$

⁴We note that a similar expression has been obtained by Carlitz and Willey[4] in the case of the trajectory $-\kappa U = \ln(-\kappa V_{cl}(U))$. When defining $-\kappa v'_L = \ln(-\kappa V')$ for $V' < 0$, their result reads

$$C_{DF,CW}^{+/-}(U, v'_L) = \left[\frac{\kappa^2}{8\pi} \frac{1}{\sinh^2(\frac{\kappa(U - v'_L - i\epsilon)}{2})} \right]^2. \quad (37)$$

as expected from Eq.(32) and the correspondence between accelerated systems described in Rindler coordinates and inertial systems in a thermal bath.

Moreover, when $g(\tau)$ is a constant, we recover that the only differences between the two-point functions obtained in the Davies-Fulling model and in the Lagrangian model concern the g_0^2 pre-factor and the additional proper time derivative. In fact these relations are generic since they directly follow from the fact that, when $g(\tau)$ is a constant, the scattering amplitudes of Eqs.(6) and (7) are proportional to the Bogoliubov coefficients obtained in the Davies-Fulling model, see Eqs.(33) in [16].

2.2.3 Other correlations on the right of an accelerated mirror

When the mirror is uniformly accelerated, in addition to the “classical” scattering analyzed above, there exists three other sectors since both \mathcal{J}^- and \mathcal{J}^+ cover two Rindler patches.

Let us first examine the trivial correlations between \mathcal{J}^- and \mathcal{J}^+ . They are obtained for $U < 0$ and any V' , *i.e.* below the past horizon of the mirror, see Fig.3. Non surprisingly, these correlations identically vanish. Indeed, causality tells us that these correlations are equal to the (null) correlations between U and V vacuum configurations evaluated on \mathcal{J}^-

$$C^{+/-}(U < 0, V') = C^{-/-}(U < 0, V') = 0 . \quad (45)$$

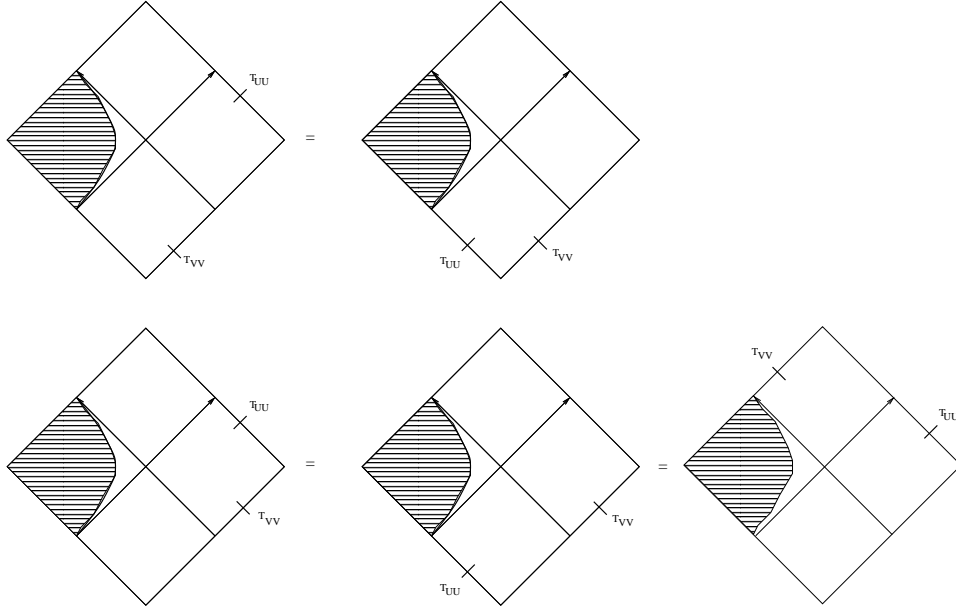


Figure 3: In these Penrose diagrams, we show the equivalence between the correlations between \mathcal{J}_R^- and \mathcal{J}_R^+ with $U < 0$ and those between \mathcal{J}_R^- and \mathcal{J}_L^- . Since the latter identically vanish, so do the formers. This is also the case in the fifth diagram which represents U/V correlations on \mathcal{J}^+ when $U < 0$, see Sec. 2.3.

The last and most interesting case corresponds to the correlations between $U > 0$ and $V' > 0$. In the Davies-Fulling model, the correlation function is again given by Eq.(34). Indeed, the Wightman function given in Eq.(25) is valid for any V' . Then, given the Rindler coordinate on the other side of the horizon $V = 0$,

$$v'_R = \frac{1}{a} \ln(aV') , \quad (46)$$

one can rewrite the correlation function in the following way

$$C_{DF,unif.acc.}^{+/-}(u_L, v'_R) = \left[\frac{a^2}{8\pi \cosh^2\left(\frac{a}{2}(v'_R + u_L)\right)} \right]^2. \quad (47)$$

This result can be obtained by analytically continuing Eq.(34) according to $V' \rightarrow V'e^{-i\pi}$. It is the $i\epsilon$ prescription which specifies that the continuation should be performed in the lower plane. In terms of Rindler coordinates, it amounts to apply $v_L \rightarrow -v_R + i\pi/a$ in Eq.(36). This analytical continuation also applies to the Unruh modes [3] and follows from the fact that Minkowski vacuum contains positive frequencies only.⁵

In the interacting model, since $V' > 0$ lies in a disconnected region for the trajectory, the field expressed at this point commutes with the Lagrangian. Hence, one has

$$\begin{aligned} F(U > 0, V' > 0) &= G(U > 0, V' > 0) \\ &= \frac{-2ig_0a^4}{(16\pi)^2} e^{-a(v'_R + u_L)} \int_{-\infty}^{+\infty} d\tau g(\tau) \frac{1}{\sinh^2\left(\frac{a(u_L - \tau - i\epsilon)}{2}\right)} \frac{1}{\cosh^2\left(\frac{a(v'_R + \tau - i\epsilon)}{2}\right)}. \end{aligned} \quad (49)$$

Thus, Eq.(23) gives

$$C_{unif.acc.}^{+/-}(U > 0, V' > 0) = 4 (\text{Re}[F(U > 0, V' > 0)])^2, \quad (50)$$

which is real, as $C_{DF}^{+/-}$ in Eq.(47). The corresponding Rindler two-point function is given by

$$C_{unif.acc.}^{+/-}(u_L, v'_R) = \frac{g_0^2}{(4\pi)^2} \left[\frac{a^2}{4} \partial_{u_L} \left(\frac{g(u_L)}{\cosh^2\left(\frac{a}{2}(u_L + v'_R)\right)} \right) \right]^2. \quad (51)$$

It is interesting to notice that Eq.(51) could have been obtained in two different ways. On one hand, Eq.(51) follows from Eq.(47) by applying the generic relations between the two models. On the other hand, Eq.(51) could have been obtained from Eq.(44) because the (regularized) scattering amplitudes, see Eq.(108), obey crossing symmetry which follows from the stability of Minkowski vacuum and which allows to deform the integral over ω' so as to obtain Eq.(49).

Since Eqs.(47) and (51) do not concern the classical reflexion on the mirror, they never diverge. In fact, they are expressed only in terms of pair creation coefficients which decrease in the ultraviolet regime like $B_{\omega\omega'}^{VU} \sim e^{-\sqrt{\omega\omega'}/a}$. Nevertheless, they are peaked around $u_L + v'_R = 0$ with a spread governed by $1/a$. This locus corresponds to $V = -V_{cl}(U)$. This maximum indicates that the configurations which give rise to the “partner” of an outgoing quantum, found for $U > 0$, are vacuum configurations which are symmetrically distributed on the other side of the horizon, see Fig.4.

⁵This continuation also applies for the trajectory $\kappa U = -\ln(-\kappa V)$ for $V < 0$. When defining $\kappa v'_R = \ln(\kappa V')$ for $V' > 0$, Carlitz and Willey noticed that

$$C_{DF,CW}^{+/-}(U, v'_R) = \left[\frac{\kappa^2}{8\pi \cosh^2\left(\frac{\kappa(U + v'_R)}{2}\right)} \right]^2 \quad (48)$$

is the analytical continuation of Eq.(37).

This result is somehow paradoxical in the case of a uniformly accelerated mirror because it indicates that pairs are steadily produced (in terms of the proper time) in a domain where the mean flux $\langle T_{UU} \rangle$ vanishes (Instead, the steady character of Eq.(48) causes no surprise since the corresponding $\langle T_{UU} \rangle$ is thermal and constant[4]). To clarify the situation, we shall analyze in Sec. 3 the correlations between U and V configurations by using the alternative method based on the conditional value of $T_{\mu\nu}$. Before doing so, it is interesting to analyze the correlations between \mathcal{J}_R^+ and \mathcal{J}_L^+ .

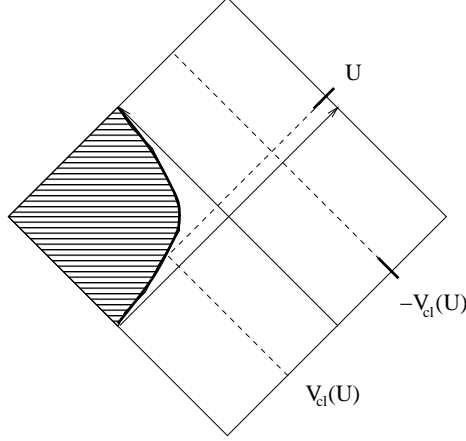


Figure 4: This diagram illustrates the fact that the correlations between T_{UU} and T_{VV} when $U > 0$ and $V' > 0$ are peaked around $V = -V_{cl}(U)$, that is, on the other side of the horizon at $V = 0$, and symmetrically with respect to the locus of classical reflection given by $V = V_{cl}(U)$, see Eqs(47) and (51). The diagram also illustrates the fact that, by causality, $C^{+/+}$ of Eq.(59) is equal $C^{+/-}$ of Eq.(47) when $U > 0$ and $V' > 0$.

2.3 The correlations on \mathcal{J}^+

We now look at the U/V correlations on \mathcal{J}^+ , that is

$$C^{+/+}(U, V') \equiv \langle 0 | T_{UU}(U, V = +\infty) T_{VV}(U' = +\infty, V') | 0 \rangle_c. \quad (52)$$

In the interacting picture, the two-point function reads

$$C^{+/+}(U, V') = \langle 0 | e^{-iL_{int}} T_{UU}(U) T_{VV}(V') e^{iL_{int}} | 0 \rangle_c. \quad (53)$$

To second order in g_0 , we get

$$\begin{aligned} C^{+/+}(U, V') &= \langle 0 | L_{int} T_{UU} T_{VV} L_{int} | 0 \rangle_c \\ &\quad - \text{Re} [\langle 0 | T_{UU} T_{VV} L_{int} L_{int} | 0 \rangle_c]. \end{aligned} \quad (54)$$

Notice that the second term behaves like the second term in the mean flux, see Eq.(10): it vanishes when integrated over V' (or U). Moreover, when applied to an accelerated mirror it leads to a vanishing $C^{+/+}(U, V')$ (as the second term in Eq.(10) leads to a vanishing mean flux) when the coupling is constant and when U and V' are both in the causal future of the mirror.

Eq.(54) guarantees that $C^{+/+}$ is real and only depends on F defined in Eq.(19):

$$C^{+/+}(U, V') = (F^*(U, V') + F(U, V'))^2 = 4 (\text{Re}[F(U, V')])^2 . \quad (55)$$

Since it arises only from pair creation amplitudes, this guarantees that it is finite, as for Eq.(51). When the trajectory is inertial, Eq.(30) gives

$$C_{inert}^{+/+}(U, V') = \frac{g_0^2}{(4\pi)^2} \left[\partial_U \left(\frac{g(U)}{(U - V')^2} \right) + \partial_{V'} \left(\frac{g(V')}{(V' - U)^2} \right) \right]^2 . \quad (56)$$

When $g(t)$ is constant, $C_{inert}^{+/+}$ vanishes for every couple of points (U, V') . This is as it should be: inertial systems do not radiate when their coupling to the radiation field is constant. Moreover, when $g(t)$ varies, Eq.(56) is finite even in the coincidence image limit, that is $V' \rightarrow V_{cl}(U)$. In this limit, one gets

$$\lim_{V' \rightarrow U} C_{inert}^{+/+}(U, V') = \frac{g_0^2}{(24\pi)^2} [\partial_U^3 g(U)]^2 . \quad (57)$$

This equation should be added to Eq.(95) in [16] which gives the mean flux emitted by this inertial mirror.

When applied to a uniformly accelerated mirror, Eq.(53) depends on the sign of U and V' . By causality, in three of the four cases, $C^{+/+}$ can be expressed in terms of the correlation functions previously computed between \mathcal{J}^- and \mathcal{J}^+ . The fourth case is the analog of the configurations studied in the inertial case, Eq.(56). We first discuss the three other cases.

Since the $V' > 0$ part of \mathcal{J}_L^+ is causally disconnected from the trajectory, one has

$$C_{unif.acc.}^{+/+}(U < 0, V' > 0) = C_{unif.acc.}^{+/-}(U < 0, V' > 0) = 0 , \quad (58)$$

according to Eq.(45), see Fig.3, and

$$C_{unif.acc.}^{+/+}(U > 0, V' > 0) = C_{unif.acc.}^{+/-}(U > 0, V' > 0) , \quad (59)$$

given in Eq.(51), see Fig.4. Moreover, the $U < 0$ part \mathcal{J}_R^+ is also disconnected from the trajectory. Hence, one obtains

$$C_{unif.acc.}^{+/+}(U < 0, V' < 0) = C_{unif.acc.}^{+/-}(V' < 0, U < 0) . \quad (60)$$

By direct evaluation, one gets

$$C_{unif.acc.}^{+/-}(v'_L, u_R) = C_{unif.acc.}^{+/-}(u_L = v'_L, v'_R = u_R) , \quad (61)$$

where $-aU = e^{-au_R}$ and $-aV' = e^{-av'_L}$ and where the r.h.s. is given by Eq.(51).

The last and most interesting correlations are encountered when the supports of T_{UU} and T_{VV} are both in the future of the trajectory, *i.e.* for $U > 0$ and $V' < 0$. In the Davies-Fulling model, the two-point function is identically zero since the mirror is perfectly reflecting. Instead, the Lagrangian model provides a non-vanishing result. In terms of Rindler coordinates, Eqs.(42) and (55) gives

$$C_{unif.acc.}^{+/+}(u_L, v'_L) = \frac{g_0^2}{(4\pi)^2} \left[\partial_{u_L} \left(\frac{g(u_L)}{\frac{4}{a^2} \sinh^2(\frac{a}{2}(u_L - v'_L))} \right) + \partial_{v'_L} \left(\frac{g(v'_L)}{\frac{4}{a^2} \sinh^2(\frac{a}{2}(v'_L - u_L))} \right) \right]^2 . \quad (62)$$

Eqs.(56) and (62) enjoy similar properties. Indeed, $C_{unif.acc.}^{+/+}$ also vanishes when g is constant and when $g(\tau)$ varies, it is finite in the coincidence image limit

$$\lim_{v'_L \rightarrow u_L} C_{unif.acc.}^{+/+}(u_L, v'_L) = \frac{g_0^2}{(24\pi)^2} [(\partial_{u_L}^3 - a^2 \partial_{u_L})g(u_L)]^2. \quad (63)$$

Eq.(62) is an illustration of Grove's theorem [29, 3] which states that the fluxes emitted by a uniformly accelerated system behave like those emitted by the same system when it is inertial and in a thermal bath. Indeed, Eq.(62) follows from Eq.(56) by replacing Minkowski coordinates by Rindler ones and the vacuum two-point function by its thermal expression. One has only transient effects when the switching function varies. This is illustrated in Fig.5 where one sees that the two-point function possesses two peaks localized in transients.

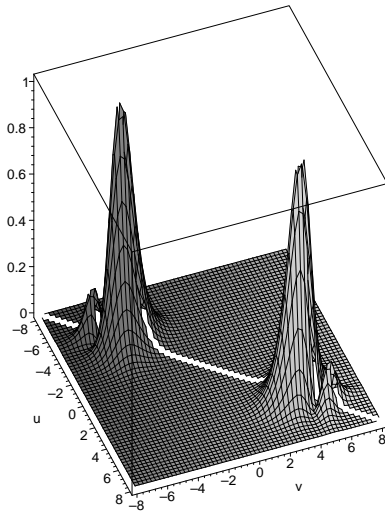


Figure 5: $C_{unif.acc.}^{+/+}(u_L, v_L)$ when $g(\tau)$ is given by Eq.(104) and in arbitrary units. We choose $a = 1$ and $\ln \eta = -4$. The two peaks are located for $u_L = v_L \simeq \pm(\ln 2\eta)$, when the coupling is switched on and off, see Eq.(105).

In brief, the lessons we obtained from the analysis of the two-point function are the following, see the two tables below. For an accelerated mirror, the mean flux and the two-point function in causal contact, Eq.(62), are concentrated in the transients. Hence they both vanish in the limit $g(\tau) \rightarrow const.$ and in the Davies-Fulling model. However, this is not the case for Eq.(59). Moreover, since the latter is a function of $u_L + v'_R$ (*i.e.* $\tau - \tau'$) when $g(\tau)$ is a constant, this indicates a steady production of particles. To understand the origin of this discrepancy, we shall analyze in the next Section matrix elements of $T_{\mu\nu}$ which contain more information than the two-point function.

3 Quantum correlations revealed by “post-selection”

To further analyze the correlations we now apply a second method. Two different situations are considered.

In Sec. 3.1, the selection of the final configurations on \mathcal{J}^+ (hereafter called “post-selection” to conform ourselves to the jargon) is imposed from the outset. This method

$\left(\frac{16\pi}{a^2 g_0}\right)^2 C_{unif.acc.}^{+/-}(u, v')$	$U = -\frac{1}{a}e^{-au_R} < 0$	$U = \frac{1}{a}e^{au_L} > 0$
$V' = -\frac{1}{a}e^{-av'_L} < 0$	0	$\left[\partial_{u_L} \left(\frac{g(u_L)}{\sinh^2(\frac{a}{2}(u_L - v'_L))} \right)\right]^2$
$V' = \frac{1}{a}e^{av'_R} > 0$	0	$\left[\partial_{u_L} \left(\frac{g(u_L)}{\cosh^2(\frac{a}{2}(u_L + v'_R))} \right)\right]^2$

Table 1: The two-point functions between \mathcal{J}^- and \mathcal{J}^+ for a uniformly accelerated mirror in L . They are of three different results. $C^{+/-}$ identically vanishes for $U < 0$, since this corresponds to causally disconnected outgoing fluxes. On the contrary, $C^{+/-}$ diverges in the coincidence image limit, for $V' = V_{cl}(U)$. Finally, $C^{+/-}$ is finite in the remaining case which corresponds to the correlations between a U quantum and its V -partner, see Fig.4.

$\left(\frac{16\pi}{a^2 g_0}\right)^2 C_{unif.acc.}^{+/+}(u, v')$	$U = -\frac{1}{a}e^{-au_R} < 0$	$U = \frac{1}{a}e^{au_L} > 0$
$V' = -\frac{1}{a}e^{-av'_L} < 0$	$\left[\partial_{v'_L} \left(\frac{g(v'_L)}{\cosh^2(\frac{a}{2}(u_R + v'_L))} \right)\right]^2$	$\left[\partial_{u_L} \left(\frac{g(u_L)}{\sinh^2(\frac{a}{2}(u_L - v'_L))} \right) + \{u_L \leftrightarrow v'_L\}\right]^2$
$V' = \frac{1}{a}e^{av'_R} > 0$	0	$\left[\partial_{u_L} \left(\frac{g(u_L)}{\cosh^2(\frac{a}{2}(u_L + v'_R))} \right)\right]^2$

Table 2: The two-point functions on \mathcal{J}^+ for a uniformly accelerated mirror in L . $C^{+/+}$ vanishes for disconnected configurations $V' > 0$ and $U < 0$. In the three other cases it is finite everywhere as it results from pair creation amplitudes only.

is straightforward and leads to the answer we are searching, namely to identify why $C^{+/-}$ in Eq.(62) vanishes whereas $C^{+/+}$ of Eq.(59) indicates a steady pair creation rate.

In Sec. 3.2 we shall nevertheless present another way to perform the post-selection: we shall couple the radiation field to an additional quantum device and then post-select the state of the latter. The justification of this second approach is that the resulting expression for the conditional value of $T_{\mu\nu}$ will establish a clear relation between the traditional approach based on the two-point function and the unusual one based on the conditional values. We shall see that the conditional value is a kind of point-split and smeared version of the two-point function previously analyzed.

3.1 The conditional value of $T_{\mu\nu}$

To compute the conditional value of an operator, one first chooses some final configurations on \mathcal{J}^+ (in our case, a wave packet representing a particle emitted toward \mathcal{J}_R^+). This defines a projector \mathbf{P} in Fock space. Then, in the Heisenberg picture, the corresponding conditional values of $T_{\mu\nu}$ reads

$$\langle T_{\mu\nu} \rangle_{\mathbf{P}} \equiv \frac{\langle 0 | T_{\mu\nu} \mathbf{P} | 0 \rangle}{\langle 0 | \mathbf{P} | 0 \rangle} = \langle 0 | T_{\mu\nu} | 0 \rangle + \frac{\langle 0 | T_{\mu\nu} \mathbf{P} | 0 \rangle_c}{\langle 0 | \mathbf{P} | 0 \rangle}. \quad (64)$$

For further details concerning the physical meaning of the (connected) matrix element of $T_{\mu\nu}$, we refer to [14].

3.1.1 The choice of the wave packet

To obtain simple expressions for $\langle T_{\mu\nu} \rangle_{\mathbf{P}}$ in the case of a uniformly accelerated mirror, we shall post-select a one-particle state described by a superposition of “Unruh” [30] modes $\tilde{\varphi}_\lambda^U$. We recall that the Unruh modes are eigenmodes with respect to the proper time τ evaluated along the trajectory and superpositions of positive frequency Minkowski modes, for further details see [30, 17]. We choose to perform the post-selection on \mathcal{J}_R^+ , in causal contact with the mirror, *i.e.* for $U > 0$, see Fig.2. The selected state is thus of the form

$$|\Psi\rangle = \int_{-\infty}^{+\infty} d\lambda f^*(\lambda; \bar{\lambda}, \sigma, \bar{u}_L) \tilde{a}_\lambda^{U\dagger} |0\rangle^{U,part}, \quad (65)$$

where $|0\rangle^{U,part}$ is the vacuum with respect to U particles and $\tilde{a}_\lambda^{U\dagger}$ is the creation operator of a Unruh U -particle. The function f is normalized as follows

$$\langle \Psi | \Psi \rangle = \int_{-\infty}^{+\infty} d\lambda \left| f(\lambda; \bar{\lambda}, \sigma, \bar{u}_L) \right|^2 = 1. \quad (66)$$

We have chosen to select this state for two reasons. First we want to detect a particle which is produced by the scattering on the mirror, *i.e.* we want $|\Psi\rangle$ to be orthogonal to the initial state, the Minkowski vacuum $|0\rangle$. This requirement excludes to work with Rindler quanta since they are present in this state. The second reason is obvious, the stationarity of the scattering is expressed in term of eigenmodes of $i\partial_\tau = \lambda$ which is a Rindler frequency.

It is also important to mention that $|\Psi\rangle$ does not fully specify the state on \mathcal{J}^+ . Indeed, since we are interested in determining the partner of $|\Psi\rangle$, we do not specify what is the state of particles emitted to \mathcal{J}_L^+ nor the anti-particle states. Therefore $|\Psi\rangle$ defines a projector $\mathbf{P}_{\bar{\lambda},\sigma}^{\bar{u}_L}$ (in the sense that $\mathbf{P}_{\bar{\lambda},\sigma}^{\bar{u}_L} \mathbf{P}_{\bar{\lambda},\sigma}^{\bar{u}_L} = \mathbf{P}_{\bar{\lambda},\sigma}^{\bar{u}_L}$) which only determines the U -particle sector:

$$\mathbf{P}_{\bar{\lambda},\sigma}^{\bar{u}_L} \equiv |\Psi\rangle\langle\Psi| \otimes \mathbf{1}[\tilde{a}^V, \tilde{b}^U, \tilde{b}^V], \quad (67)$$

since $\mathbf{1}[\tilde{a}^V, \tilde{b}^U, \tilde{b}^V]$ is the identity operator for the particle states on \mathcal{J}_L^+ and the anti-particle states.

To obtain analytical expressions, we choose the function f to be

$$f(\lambda; \bar{\lambda}, \sigma, \bar{u}_L) \equiv \frac{e^{i\lambda\bar{u}_L}}{\sqrt{4\pi\lambda}(e^{2\pi\lambda/a} - 1)} \frac{e^{-(\lambda-\bar{\lambda})^2/2\sigma^2}}{\sqrt{2\pi\sigma}} \times \lambda \sinh^2(\pi\lambda/a) \times \mathcal{N}[\bar{\lambda}, \sigma]. \quad (68)$$

The first factor corresponds to a Unruh quantum centered around $u_L = \bar{u}_L$ with a mean frequency given by $\bar{\lambda}$. The second factor has been chosen so as to obtain analytical expressions for the conditional values and the third factor ensures that $|\Psi\rangle$ is normalized according to Eq.(66). Since we want to detect the particle well localized around \bar{u}_L , *i.e.* around $a\bar{U} = e^{a\bar{u}_L} > 0$, $\bar{\lambda}$ must obey

$$\bar{\lambda} < 0 \text{ and } |\bar{\lambda}|/a \gg 1. \quad (69)$$

The first condition arises from the fact that Unruh modes of negative Rindler frequency live mainly in the L sector. The second condition guarantees that the second peak of the wave packet found around $U = -\bar{U} < 0$ is negligible. We recall that wave packets built

with Unruh modes possess two peaks. The relative weights of their norms is the thermal factor $e^{-2\pi|\bar{\lambda}|/a}$ encoding the Unruh effect.

Before computing the conditional value of the flux, one inquires into the Minkowski frequency content of $|\Psi\rangle$. To this end, we compute the probability to find a one-particle state of Minkowski frequency ω

$$\langle 0|a_\omega^U \mathbf{P}_{\bar{\lambda},\sigma}^{\bar{u}_L} a_\omega^{U\dagger}|0\rangle = \left| \int_{-\infty}^{+\infty} d\lambda f^*(\lambda; \bar{\lambda}, \sigma, \bar{u}_L) \gamma_{\lambda\omega} \right|^2, \quad (70)$$

where

$$\begin{aligned} \gamma_{\lambda\omega} &\equiv \langle 0|a_\omega^U \tilde{a}_\lambda^{U\dagger}|0\rangle \\ &= \frac{\Gamma(i\lambda/a)}{\sqrt{\frac{a\pi}{\lambda \sinh(\pi\lambda/a)}}} \left(\frac{\omega}{a}\right)^{-i\lambda/a} \frac{1}{\sqrt{2\pi\omega a}}. \end{aligned} \quad (71)$$

In the limit $|\bar{\lambda}|/a \gg 1$ the stationary phase condition gives

$$\lambda_{sp} = \bar{\lambda} - i \frac{\sigma^2}{a} \ln\left(\frac{-\omega}{\lambda_{sp} e^{-a\bar{u}_L}}\right). \quad (72)$$

The norm of the overlap is maximum when the imaginary part of λ_{sp} vanishes. Thus $|\Psi\rangle$ is made of Minkowski frequencies centered around $\bar{\omega} = |\bar{\lambda}|e^{-a\bar{u}_L}$. The spread in ω is $\sigma e^{-a\bar{u}_L}$. When $\sigma < a$, our wave packet is thus well-peaked both in Minkowski frequencies and in space-time.

3.1.2 The conditional values of the fluxes

To obtain the connected part of the conditional value, we first compute the denominator of the second term in Eq.(64) which gives the probability to detect our chosen quantum. To the second order in g_0 , it is given by $\langle 0|L_{int} \mathbf{P}_{\bar{\lambda},\sigma}^{\bar{u}_L} L_{int}|0\rangle$. Since $|\Psi\rangle$ is expressed in terms of Unruh quanta, it is appropriate to re-express the transition amplitudes in terms of these rather than Minkowski quanta as in Eqs.(6) and (7). The resulting amplitudes will be noted $\tilde{B}_{\lambda\lambda'}$. We have suppress the upper indices U, V because they are all equal. Moreover they simply depend on the Rindler Fourier components of $g(\tau)$:

$$\begin{aligned} \tilde{B}_{\lambda\lambda'} &\equiv \langle 0|\tilde{a}_\lambda^U \tilde{b}_{\lambda'}^V iL_{int}|0\rangle = \langle 0|\tilde{a}_\lambda^U \tilde{b}_{\lambda'}^U iL_{int}|0\rangle \\ &= ig_0 \frac{\lambda\lambda'}{\sqrt{\lambda(e^{2\pi\lambda/a} - 1)\lambda'(e^{2\pi\lambda'/a} - 1)}} g_{\lambda+\lambda'}^*. \end{aligned} \quad (73)$$

Then the probability reads

$$\langle 0|L_{int} \mathbf{P}_{\bar{\lambda},\sigma}^{\bar{u}_L} L_{int}|0\rangle = 2 \iint \int_{-\infty}^{+\infty} d\lambda d\lambda' d\mu \tilde{B}_{\lambda\mu}^* \tilde{B}_{\lambda'\mu} f^*(\lambda) f(\lambda'). \quad (74)$$

We must now verify that the matrix elements computed with $\tilde{B}_{\lambda\lambda'}$ converge. We remind the reader that when computing the *average* value of the fluxes, convergence was provided by the asymptotic decreasing of $g(\tau)$, namely $g(\tau) \rightarrow 0$ faster than $e^{-a|\tau|}$, see [17]. In the case of *conditional* values, a UV frequency cut-off can be provided by $f(\lambda)$ which

characterizes the post-selected wave packet. When this is the case, one can safely consider the limit of constant coupling: $g(\tau) = 1$. In this limit, we get

$$\tilde{B}_{\lambda\lambda'} = -\frac{ig_0}{2} \frac{\lambda}{\sinh(\pi\lambda/a)} \delta(\lambda + \lambda') . \quad (75)$$

When using f given in Eq.(68), the Gaussian weight guarantees that all expressions are well-defined in the UV. Moreover, Eq.(68) leads to analytical expressions for the probability and the conditional values of the flux. However, since these expressions are rather complicated, we shall present only their behavior in the limit $|\bar{\lambda}|/a \gg \sigma^2/a^2$ in addition to Eq.(69) (This condition means that we work with well peaked wave packets in λ). In this limit, the probability reads

$$\langle 0 | L_{int} \mathbf{P}_{\bar{\lambda}, \sigma}^{\bar{u}_L} L_{int} | 0 \rangle = 2g_0^2 \bar{\lambda}^2 e^{-2\pi|\bar{\lambda}|/a} , \quad (76)$$

which is independent of the value of \bar{u}_L , thereby indicating a steady production of particles weighted by the thermal factor, as expected from the general analysis of Appendix C in [14]. (The corrections to Eq.(76) and the following equations are $O(\sigma/\bar{\lambda})$).

The connected part of the conditional values of the flux is

$$\langle T_{\mu\nu} \rangle_{\mathbf{P}_c} = \langle T_{\mu\nu} \rangle_{\mathbf{P}}^{p.s.} + \langle T_{\mu\nu} \rangle_{\mathbf{P}}^{partner} , \quad (77)$$

where $\langle T_{\mu\nu} \rangle_{\mathbf{P}}^{p.s.}$ is the flux carried by the post-selected particle and $\langle T_{\mu\nu} \rangle_{\mathbf{P}}^{partner}$, that carried by its partner. When \mathbf{P} is given by Eq.(67), $\langle T_{\mu\nu} \rangle_{\mathbf{P}}^{p.s.}$ is purely outgoing and consists in matrix elements of $\tilde{a}^U \tilde{a}^{U\dagger}$ and $\tilde{a}^U \tilde{b}^U$. It possesses two maxima for $U = \bar{U}$ and $U = -\bar{U}$, which are related to the probability of measuring the position of the post-selected wave packet. Moreover, as for Hawking quanta emerging from a black hole, it is complex [14].

The “partner” term of Eq.(77) is more interesting. To second order in g_0 , it is given by

$$\langle T_{\mu\nu} \rangle_{\mathbf{P}}^{partner} = \frac{\langle 0 | L_{int} T_{\mu\nu} \mathbf{P}_{\bar{\lambda}, \sigma}^{\bar{u}_L} L_{int} | 0 \rangle}{\langle 0 | L_{int} \mathbf{P}_{\bar{\lambda}, \sigma}^{\bar{u}_L} L_{int} | 0 \rangle} \quad (78)$$

Unlike the *p.s.* term, $\langle T_{\mu\nu} \rangle_{\mathbf{P}}^{partner}$ contains both U and V fluxes. Moreover, since the scattering amplitudes of Eq.(73) are identical for U and V modes, one has $\langle T_{UU}(U) \rangle_{\mathbf{P}}^{partner} = \langle T_{VV}(V = -U) \rangle_{\mathbf{P}}^{partner}$. Explicitly one gets

$$\langle T_{VV} \rangle_{\mathbf{P}}^{partner} = \frac{2 \left| \int \int_{-\infty}^{+\infty} d\lambda d\lambda' \tilde{B}_{\lambda\lambda'} f(\lambda) \partial_V \tilde{\varphi}_{\lambda'}^V \right|^2}{\langle 0 | L_{int} \mathbf{P}_{\bar{\lambda}, \sigma}^{\bar{u}_L} L_{int} | 0 \rangle} , \quad (79)$$

where $\tilde{\varphi}_{\lambda}$ is the mode associated with the Unruh operator \tilde{a}_{λ} . This part of the conditional flux consists only in matrix elements of $\tilde{b}^V \tilde{b}^{V\dagger}$. Having post-selected a wave packet made with \tilde{a} only, this guarantees that the “partner” term is real.

In the right quadrant, for $V > 0$, the Rindler flux is

$$\langle T_{vv}(v_R) \rangle_{\mathbf{P}}^{partner} = \frac{\sigma|\bar{\lambda}|}{\sqrt{\pi}} e^{-(\bar{u}_L + v_R)^2 \sigma^2} . \quad (80)$$

In this, we find a behavior very similar to that of $C^{+/+}(\bar{u}_L, v_R)$ of Eq.(59): $C^{+/+}$ also exhibits a constant maximum for $v_R = -\bar{u}_L$, see Table 2. The width is here given by $1/\sigma$ instead of $1/a$ as in Eq.(59), because of our choice of the window function $f(\lambda)$ in Eq.(68). The similarity of $\langle T_{vv}(v_R) \rangle_{\mathbf{P}}^{partner}$ when having post-selected a U -quantum at \bar{u}_L , with $C^{+/+}(\bar{u}_L, v_R)$ should cause no surprise: the first term of Eq.(54) is dominant and T_{UU} acts in it as \mathbf{P} does it in Eq.(78).

However the correspondence with the two-point function is lost when computing the conditional value in the left quadrant, for $V < 0$. Whereas $C^{+/+}(u_L, v'_L)$ vanishes for $g(\tau) = 1$, we obtain

$$\langle T_{vv}(v_L) \rangle_{\mathbf{P}}^{partner} = \frac{\sigma|\bar{\lambda}|}{\sqrt{\pi}} e^{-(\bar{u}_L - v_L)^2 \sigma^2} e^{-2\pi|\bar{\lambda}|/a}, \quad (81)$$

which is smaller than Eq.(80) by the thermal factor $e^{-2\pi|\bar{\lambda}|/a}$. The origin of this loss is as follows. In Eq.(53) the post-selection induced by $T_{uu}(u_L)$ is strictly confined in the L quadrant. Hence it is insensitive to the transients (located on $U = 0^+$ in the limit $g \rightarrow const.$) which contain all the emitted particles. On the contrary, the post-selection carried by $|\Psi\rangle$ specifies that one Unruh quantum be present on \mathcal{J}_R^+ . This prescription is sensitive to the particle content of the transients. In fact, *all* wave packets made with only positive Minkowski frequency modes are sensitive to these transients, since there exists *no* such wave packet which can vanish in a Rindler quadrant.⁶ Had we post-selected a superposition of L Rindler quanta only, we would have found that $\langle T_{vv}(v_L) \rangle_{\mathbf{P}}^{partner}$ identically vanished, exactly like $C^{+/+}$ of Eq.(62). The origin of this null result can be traced back to Eq.(54). When one post-selects only L Rindler quanta, the contribution of the second (interfering) term cancels that of the first term. Instead when imposing that a superposition of Minkowski quanta be found on \mathcal{J}^+ , the second term vanishes since the post-selected state is orthogonal to Minkowski vacuum.

We learned from this analysis that, being local in U , $C^{+/+}(u_L, v'_L)$ is an extremely coherent object whose vanishing results from fine tuned interferences. The slightest modification of the scattering, *e.g.* recoil effects[15, 27] or switching off effects, would break these interferences. This leads to a non-vanishing result whose content of Unruh quanta tells us that the pair creation process is stationary. In conclusion, the two-point correlation function vanishes because, on one hand, it probes only locally the final configurations, and, on the other, the description of the scattering is too simplified.

When looking at the partner conditional flux on \mathcal{J}_R^+ , one obtains the same expressions as in Eqs.(80) and (81) with $v \rightarrow u$

$$\langle T_{uu}(u_R) \rangle_{\mathbf{P}}^{partner} = \frac{\sigma|\bar{\lambda}|}{\sqrt{\pi}} e^{-(\bar{u}_L + u_R)^2 \sigma^2} \quad (82)$$

$$\langle T_{uu}(u_L) \rangle_{\mathbf{P}}^{partner} = \frac{\sigma|\bar{\lambda}|}{\sqrt{\pi}} e^{-(\bar{u}_L - u_L)^2 \sigma^2} e^{-2\pi|\bar{\lambda}|/a}. \quad (83)$$

From these, one sees that the U partner of a U post-selected wave packet lies mainly on the other side of the horizon $U = 0$, see Fig.6, and is distributed in a way which once more displays the stationarity of the process.

⁶The first explanation of the compatibility of a null mean local flux with the readings of a particle detector was made in [31]. Grove proved that the detection of particles produced by a uniformly accelerating mirror occurs only if the detector is switched on in the causal future of the transients, see also [32] for a similar observation in a slightly different context.

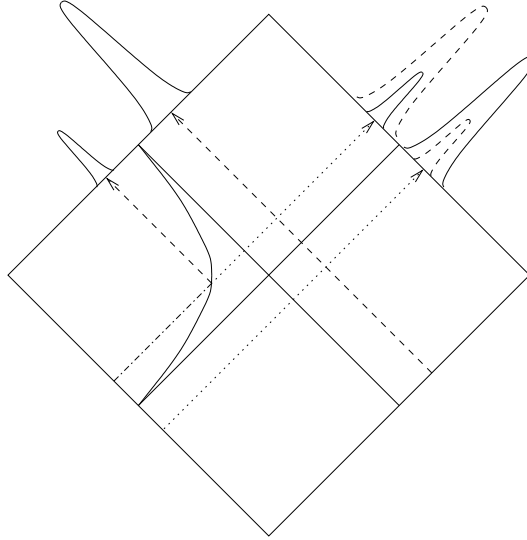


Figure 6: In this figure, the intensity profile of the post-selected wave packet is drawn as a dashed curve. The corresponding values of the partner conditional fluxes are represented by plain curves. The U/V symmetry of these fluxes is manifest. The dotted and dashed straight lines schematically represent the characteristics followed by the partner configurations.

In conclusion, we notice that the transition amplitude $B_{\lambda\lambda'}$ of Eq.(75) is unchanged if one now considers the scattering by a mirror moving in the right quadrant. Hence, one would obtain exactly the same conditional fluxes. To obtain expressions which depend on the side in which the mirror lives, one should consider time dependent coupling. This is the subject of Sec. 4. Before doing so, we shall consider another way to implement the post-selection which will reveal the relations between the conditional fluxes and the two-point function.

3.2 Post-selection by an additional quantum device

Another way to implement the post-selection is to introduce an additional quantum system coupled to the field on the right of the mirror. In what follows, we shall use an inertial two-level atom[2, 3] positioned at $z = \text{const}$, on the right of the mirror. The transitions of the two-level atom are described by the lowering operator $A(t) = e^{-imt} A$ and its hermitian conjugate. Here, m is the energy gap between its ground ($|-\rangle$) and excited state ($|+\rangle$). One has $A|-\rangle = 0$ and $A|+\rangle = |-\rangle$.

To make contact with Sec. 3.1, we couple the detector only to U quanta. This is achieved by the following action

$$L_A = -f_0 \int dU f(U) \left(\partial_U \Phi^U \dagger A e^{-imU} + \partial_U \Phi^U A^\dagger e^{imU} \right) . \quad (84)$$

Here, f_0 is the coupling constant and $f(t)$ is a real function which governs how the interaction is turned off and on.

Instead of Eq.(64), we consider the value of the energy flux $\langle T_{VV} \rangle_{\mathbf{\Pi}}$ which is conditional to find the detector in its excited state at $t = +\infty$ when the initial state is $|0\rangle \otimes |-\rangle$. The post-selection is imposed by applying the projector $\mathbf{\Pi} = |+\rangle\langle +|$ at $t = +\infty$. To second

order in f_0 , in the Heisenberg representation for the evolution governed by L_{int} , we obtain

$$\langle T_{VV} \rangle_{\mathbf{\Pi}} = \langle 0|T_{VV}|0\rangle + \frac{\langle 0|T_{VV}\tilde{\mathbf{\Pi}}|0\rangle_c}{\langle 0|\tilde{\mathbf{\Pi}}|0\rangle}, \quad (85)$$

where

$$\tilde{\mathbf{\Pi}} = \langle -|L_A \mathbf{\Pi} L_A|-\rangle = f_0^2 \int \int_{-\infty}^{+\infty} dU dU' f(U) f(U') e^{-im(U-U')} \partial_U \Phi^\dagger \partial_{U'} \Phi. \quad (86)$$

In the connected part of the conditional flux, the time ordering of L_A with L_{int} is such that L_A can always be sent on the future of the evolution operator $Te^{iL_{int}}$ since the detector is on the right of the mirror and since it responds only to U quanta. In the interacting picture, the second term in Eq.(85) is

$$\langle T_{VV} \rangle_{\mathbf{\Pi}c} = \frac{\int \int_{-\infty}^{+\infty} dU dU' f(U) f(U') e^{-im(U-U')} \langle 0|e^{-iL_{int}} \partial_U \Phi^\dagger \partial_{U'} \Phi T_{VV} e^{iL_{int}} |0\rangle_c}{(\langle 0|e^{-iL_{int}} \tilde{\mathbf{\Pi}} e^{iL_{int}} |0\rangle / f_0^2)}. \quad (87)$$

This expression should be compared to the two-point function of Eq.(53) and to the former conditional flux, see Eq.(78). Two limits can be considered. In the first limit, f is localized in space-time, *i.e.* $f(U) = \delta(U - U_0)$. In this case, the numerator gives the two-point function $C^{+/+}(U_0, V)$ of Eq.(53) whereas the denominator gives the mean value of T_{UU} . In the second limit, the Fourier transform of $f e^{-imU}$ is “local” in the energy space, *i.e.* $\tilde{f}(\omega) = \delta(\omega - m)$. In this case, the particle detector is switched on for all times and is sensitive only to Minkowski quanta of frequency m . Then, up to an overall constant, the projector $\tilde{\mathbf{\Pi}}$ reduces to the normal-ordered counting operator $\mathbf{P}_m = a_m^U{}^\dagger a_m^U$. When acting on one-particle Minkowski states, it thus acts as the projector of Eq.(67).

We have thus proved that the conditional value, Eq.(87), generalizes the notion of correlation functions as it interpolates from the local two-point function $\langle T_{UU} T_{VV} \rangle$ to the global correlation $\langle a_m^U{}^\dagger a_m^U T_{VV} \rangle$ which relies on the notion of particle. The intermediate cases correspond to smeared and point-split expressions, see [33, 34] for similar considerations on smeared correlation functions.

4 Scattering by two accelerated mirrors

In this Section, we study the scattering by two accelerating mirrors which follow symmetrical trajectories in the R and L quadrants (see Fig.9). In Sec. 4.1, using the (unregulated) Davies-Fulling model, we show that the Bogoliubov coefficients encoding pair creation identically vanish. Their vanishing arises from perfect interferences, in very much the same way of the perfect interferometer of Gerlach[24]. In Sec. 4.2, we prove that these perfect interferences are an artifact of the unregulated description in which the coupling of the mirror to the radiation field is strictly constant. In Sec. 4.3, we show how this singular regime can be approached (but never fully realized if one insists on keeping regularity) by fine-tuning the coupling constant.

4.1 The Davies-Fulling description

In the Davies-Fulling model, the mode scattered by the two mirrors is given by

$$\varphi_\omega^{scat}(U) = -\frac{e^{-i\omega V_{cl}(U)}}{\sqrt{4\pi\omega}}, \quad (88)$$

for *all* U . Indeed, the peculiarity of two symmetrical uniformly accelerated trajectories is that $V_{cl}(U) = -1/a^2 U$ is valid for all values of U (and V), as in the case of a single mirror which originates from i^- and ends in i^+ . If only one accelerated mirror was present, the support of $V_{cl}(U) = -1/a^2 U$ would have been restricted to half the real axis.

The Bogoliubov coefficient $^{L+R}\beta_{\omega\omega'}^{UV*}$ encoding pair creation is given, as usual, by the overlap between φ_{ω}^{scat} with an *out*-mode of frequency ω' , see Eq.(13). Thus one has

$$^{L+R}\beta_{\omega\omega'}^{UV*} = {}^L\beta_{\omega\omega'}^{UV*} + {}^R\beta_{\omega\omega'}^{UV*}, \quad (89)$$

where ${}^L\beta$ (${}^R\beta$) is the Bogoliubov coefficient one would obtain when considering only one mirror. Moreover since [2]

$${}^R\beta_{\omega\omega'}^{UV*} = \frac{i}{\pi a} K_1(2\sqrt{\omega\omega'}/a) = - {}^L\beta_{\omega\omega'}^{UV*}, \quad (90)$$

the total Bogoliubov coefficient $^{L+R}\beta_{\omega\omega'}^{UV}$ vanishes for all values of ω and ω' !!! Thus the total energy emitted

$$\langle H_M^V \rangle = \int_0^\infty d\omega \, \omega \int_0^\infty d\omega' \, |^{L+R}\beta_{\omega\omega'}^{UV*}|^2 \quad (91)$$

vanishes as well since it is given in terms of the square of ${}^L\beta^{UV} + {}^R\beta^{UV}$ and not in terms of the sum of their squares. Therefore, $\langle H_M^V \rangle$ vanishes because of the perfectly destructive interferences between the scattering amplitudes.

In brief, in this description, no pair is created, *i.e.* the two mirrors have no effect on the vacuum configurations, exactly like an inertial mirror. This cancellation is directly related to what Gerlach called a perfect interferometer, see Eq.(125) in [24]. It is also related to the canceling effect found by Yi[25] when considering the asymptotic radiation emitted by two accelerated (charged) black holes.

It is important to verify that this result is not due to the fact that the mirrors are perfectly reflecting. In fact, it is also obtained when using partially transmitting mirrors in constant interaction with the radiation field. This is easily verified by using the interacting model. In this case, each mirror is coupled to the field by a Lagrangian given by Eq.(1) with coupling parameter g_0^L (g_0^R) and switching function $g^L(\tau)$ ($g^R(\tau)$). The total Lagrangian is the sum $L_{int} = L^L + L^R$. Thus, using Eq.(7), to first order in g_0^L and g_0^R , the amplitude of spontaneous pair creation $^{L+R}B_{\omega\omega'}^{ij} = \langle 0|a_{\omega}^i b_{\omega'}^j, iL_{int}|0\rangle$ identically vanishes when⁷ $g_0^R = -g_0^L$ and when $g^R(\tau) = g^L(\tau) = \text{constant}$. Hence, perfect interferences do not follow from perfect reflection. As we shall now prove, they directly follow from the fact the coupling is constant.

4.2 The regulated description

In [17], we proved that the scattering by a uniformly accelerated mirror is regular only if its coupling to the radiation field decreases faster than $e^{-a|\tau|}$. (If this condition is not fulfilled, the expectation values of observables are ill-defined, *i.e.* the result might depend on the order in which the integrations are performed.)

⁷Had we taken the current $J = \Phi^\dagger i \overleftrightarrow{\partial}_\tau \Phi$ instead of $\partial\tau\Phi^\dagger\partial\tau\Phi + \partial\tau\Phi\partial\tau\Phi^\dagger$ in Eq.(1), the condition would have been $g_0^R = g_0^L$, thereby recovering the correspondence between the Lagrangian model and the Davies-Fulling one.

We shall now prove that by using *any* regular description of the scattering, the interferences leading to the vanishing of the $^{L+R}\beta$ coefficients are *inevitably* lost. To this end, it is convenient to work in a “mixed” representation, *i.e.* with one quantum characterized by a Minkowski frequency and its partner by a Unruh quantum with a given Rindler frequency, see Sec. 2.3 in [17] for detailed expressions. In this case, when using $g(\tau)$ given in Eq.(104), the spontaneous pair creation amplitude is

$$\begin{aligned} {}^{R,L}B_{\omega\lambda}^{VV} &\equiv \langle 0|a_{\omega}^V \tilde{b}_{\lambda}^U e^{iL^{R,L}}|0\rangle_c \\ &= \frac{ig_0}{\pi a} \sqrt{\frac{\omega\lambda}{1 - e^{-2\pi\lambda/a}}} (1 - is\omega/a\eta)^{-(1+i\lambda/a)/2} K_{1+i\lambda/a}(2\eta\sqrt{1 - is\omega/a\eta}) \end{aligned} \quad (92)$$

where $s = \pm 1$ for R and L respectively. One also finds that ${}^{R,L}B_{\omega\lambda}^{VU} = {}^{R,L}B_{\omega\lambda}^{VV}$. We thus see that the regulator $\eta \ll 1$, which is needed to obtain well-defined expressions, enters differently in the amplitudes of the R and L mirrors. In fact, this is necessary for causality to be respected. Hence, it is quite conceivable that the former canceling effects will be lost when ω approaches the UV scale a/η .

Let us verify this numerically by studying the integrand of the Minkowski energy:

$$\langle H_M^V \rangle^{L+R} = 4 \int_0^\infty d\omega \, \omega \int_{-\infty}^{+\infty} d\lambda \, |{}^L B_{\omega\lambda}^{VV} + {}^R B_{\omega\lambda}^{VV}|^2 = \int_0^\infty d\omega \int_{-\infty}^{+\infty} d\lambda \, h_M^{L+R}(\omega, \lambda; \eta) . \quad (93)$$

When $g_0^R = -g_0^L$ and when $g^R(\tau) = g^L(\tau)$ is given by Eq.(104), the integrand h_M^{L+R} is shown in Fig.8. One clearly sees how $g(\tau)$ plays its role. The transients associated

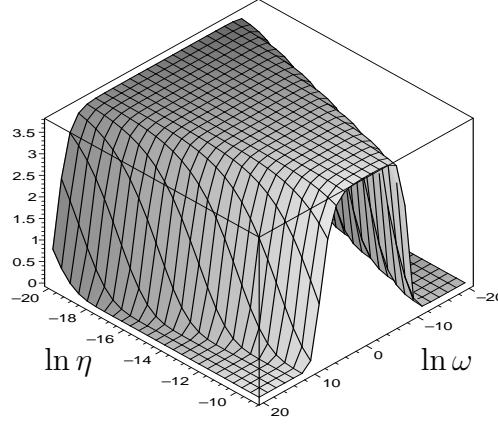


Figure 7: The integrand for one mirror $h_M^L(\omega, \lambda = 0.15a; \eta)$ in terms of $\ln\omega$ and $\ln\eta$ and in arbitrary units. One sees clearly that the contribution of Minkowski modes exhibits a “plateau” of constant height which is limited by the cut-offs $\ln\omega = \pm \ln\eta$. All the modes whose frequency belongs to this interval participate to the emitted flux.

by the switching function are different for the right and left mirrors. Hence the sum of amplitudes no longer vanishes for frequencies close to the frequency cut-offs $a\eta$ and a/η . Instead, all the modes within the plateau, *i.e.* when the interaction is almost constant, interfere destructively, as in the Davies-Fulling model. Hence, they do not participate to the total energy.

There is another simple way to prove that the two mirrors cannot interfere destructively. It follows from causality. In fact, since the two mirrors are causally disconnected, the mean flux obeys

$$\langle T_{VV}(V) \rangle^{L+R} = \langle T_{VV}(V) \rangle^L + \langle T_{VV}(V) \rangle^R , \quad (94)$$

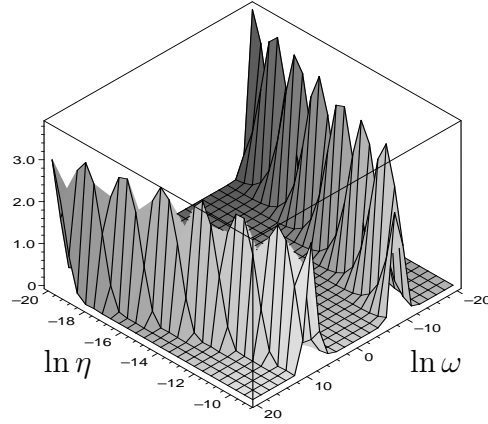


Figure 8: The integrand for the two mirrors $h_M^{L+R}(\omega, \lambda = 0.15a; \eta)$ in terms of $\ln \omega$ and $\ln \eta$ and in the same arbitrary units. Unlike for a single mirror, the “inside” modes with $a\eta \ll \omega \ll a/\eta$ no longer contribute because of the destructive interferences. However, these interferences are lost when reaching the transient frequencies $\omega = a\eta$ and $\omega = a/\eta$, thereby giving rise to a positive integrand.

since $\langle T_{VV} \rangle^{L(R)}$ vanishes identically in $R(L)$. Hence

$$\begin{aligned} \langle H_M^V \rangle^{L+R} &= \langle H_M^V \rangle^L + \langle H_M^V \rangle^R \\ &= 4 \int_0^\infty d\omega \, \omega \int_{-\infty}^{+\infty} d\lambda \left(|{}^L B_{\omega\lambda}^{VV}|^2 + |{}^R B_{\omega\lambda}^{VV}|^2 \right). \end{aligned} \quad (95)$$

This tells us that whatever are the couplings between the two mirrors and the radiation field, when causality is respected, the total energy emitted by the mirrors can be added incoherently, *i.e.*, *unlike* what was found in the Davies-Fulling model in Eq.(91). Therefore, when causality is respected, Eqs.(93) and (95) imply

$$\int_0^\infty d\omega \, \omega \int_{-\infty}^{+\infty} d\lambda \, \text{Re} \left\{ {}^R B_{\omega\lambda}^{VV} {}^L B_{\omega\lambda}^{VV*} \right\} \equiv 0. \quad (96)$$

To complete the analysis we now determine to what extent two accelerated mirrors can constitute a perfect interferometer. That is : What is the most interfering situation ?

4.3 A perfect interferometer ?

To answer this question, we use the interacting model and we parametrize the coupling constant in Fourier transform: $g^L(\tau) = \int_{-\infty}^{+\infty} d\lambda \, g_\lambda^L e^{-i\lambda\tau}$ and $g^R(\tau) = \int_{-\infty}^{+\infty} d\lambda \, g_\lambda^R e^{-i\lambda\tau}$.

Instead of studying the average value of H or T_{VV} , it is simpler to compute the probability to receive one Unruh quantum on \mathcal{J}^+ . The two accelerating mirrors would provide a perfectly interfering device if this probability vanishes. To compute it we use the projector $\mathbf{P}_{\bar{\lambda}}$ associated with the detection of a Unruh quantum on \mathcal{J}_R^+ , *i.e.* with

$$f(\lambda) = \delta(\lambda - \bar{\lambda}) \frac{e^{i\lambda\bar{u}_L}}{\sqrt{4\pi\lambda(e^{2\pi\lambda/a} - 1)}} \quad (97)$$

in Eq.(68). To the second order in the g_0 's, from Eq.(74), we get

$$\langle \mathbf{P}_{\bar{\lambda}} \rangle^{L+R} = \frac{n_{\bar{\lambda}}^2}{2\pi} \int_{-\infty}^{+\infty} d\lambda \, \lambda n_\lambda \left| g_0^L g_{\lambda+\bar{\lambda}}^{L*} + e^{\pi(\bar{\lambda}+\lambda)/a} g_0^R g_{\lambda+\bar{\lambda}}^R \right|^2. \quad (98)$$

The above expression vanishes if

$$g_0^L g_\lambda^{L*} = -e^{\pi\lambda/a} g_0^R g_\lambda^R, \quad (99)$$

for all λ . The only solution for (real) coupling functions is given by time-independent opposite real numbers : $g^L(\tau) = g^R(\tau) = 1$ and $g_0^L = -g_0^R$. Therefore, we find that one can form a perfect interferometer *if and only if* one considers two mirrors constantly in interaction with the radiation. However, the necessary condition to obtain regular Minkowski expressions is then violated.

So let us minimize the probability $\langle \mathbf{P}_\lambda \rangle^{L+R}$ by insisting that one has regular expressions, *i.e.* that both g_R and g_L decrease faster than $e^{-a|\tau|}$. For instance, take Gaussian switching functions for each mirror with *a priori* different width and center:

$$g^R(\tau) = e^{-(\tau-\tau_R)^2/2T_R^2} \quad \text{and} \quad g^L(\tau) = e^{-(\tau-\tau_L)^2/2T_L^2}. \quad (100)$$

For simplicity and from our previous analysis, we suppose that $g_0^R = -g_0^L$. We also work in the double limit of rare events $|\bar{\lambda}|/a \gg 1$ and long couplings $aT_{L,R} \gg 1$. Then the ratio of the probabilities when having one or two mirrors is

$$\frac{\langle \mathbf{P}_\lambda \rangle^{L+R}}{\langle \mathbf{P}_\lambda \rangle^L} \propto \frac{1}{a^2 T^2} (1 + \chi_1 (\tau_R + \tau_L)^2 a^2 + \chi_2 |T_R - T_L|/T_L), \quad (101)$$

where χ_1 and χ_2 are positive factors of order unity and $T = (T_R + T_L)/2$. Therefore, the minimization of the probability is reached for switching functions:

- which possess the same lapse $T_R = T_L = T$
- and which are centered symmetrically: $\tau_R + \tau_L = 0$ (see Fig.9).

When $T \rightarrow +\infty$, we get $\langle \mathbf{P}_\lambda \rangle^{R+L}/\langle \mathbf{P}_\lambda \rangle^L \rightarrow 0$, thereby approaching the perfect interferometer behavior. However, in this case, the Minkowski energy will diverge like e^{aT} .

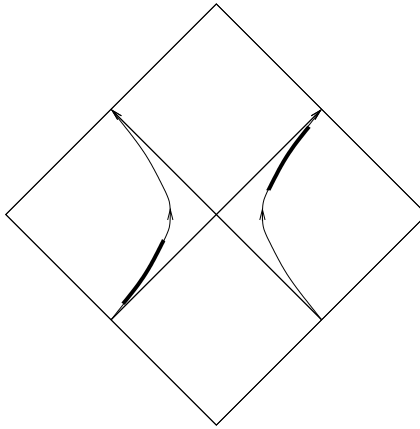


Figure 9: In this diagram, we show the trajectories followed by the two mirrors. We represent an example of a fine-tuned device by drawing thicker lines when the interactions are switched on. The centers of the thicker parts are symmetrical with respect to $U = V = 0$ and the lapses are identical.

Conclusions

We recall the main results derived in this paper. We first study the quantum correlations within the fluxes emitted by a uniformly accelerated mirror. The results, which are summarized in Tables 1 and 2, reveal how the original correlations of the vacuum are scattered by the mirror. However, this analysis is partial in that the particle content of the fluxes is not disentangled when probing the final state by local operator. This is particularly clear for $C^{+/+}$ of Eq.(62) which vanishes in the limit $g \rightarrow \text{const.}$

To complete the analysis, we then compute the conditional flux which is correlated to the detection of a given outgoing particle. These conditional fluxes are rather similar to the corresponding two-point functions. However, they differ in one respect: the second term of Eq.(54) never contributes to the conditional value whereas it does for the two-point function. In fact, it is at the origin of the vanishing of $C^{+/+}$ of Eq.(62). In addition, once its contribution is suppressed, either by a post-selection or by taking into account recoil effects, one finds that the scattering by a uniformly accelerated mirror leads to a steady production of pairs of quanta.

To further relate the usual correlation functions to the conditional values of the fluxes, we present another way to perform the post-selection so as to be able to recover the formers as limiting cases of the latter. We believe that this is quite important for the following reasons. First, when studying quantum field theory in a curved space-time[2], one loses the notion of particle which exists in Minkowski space-time. Therefore, in the absence of a unique definition of particle, it has been claimed that the only meaningful quantities are expectation values of local operators. We find this claim too restrictive in that not only mean quantities have physical meaning in quantum settings: To ask what happened when a particle detector did click, or did not click, are perfectly legitimate questions. It is therefore of importance to establish an explicit relationship between the local and the particle descriptions. This has been here achieved by introducing a particle detector, computing the conditional value of the flux, and adjusting the window function both in space-time and in the energy space, so as to generate a spectrum of matrix elements which reduce, in limiting cases, to conventional expectation values of local operators and global operators.

Secondly, the correlations of local operators are singular in the coincidence point limit and near a black hole horizon[18, 19, 20, 21, 14, 11] or in inflationary cosmology[22, 23]. It is still unclear at present how to handle these divergences when considering the gravitational back-reaction associated with the fluctuations of $T_{\mu\nu}$. It is thus also of importance to deal from the outset with point-split and smeared generalization of (ultra-local) correlation functions. Such a generalization is naturally obtained by considering the conditional fluxes as computed in Section 3.2.

Finally, to further illustrate the need of using regularized transition amplitudes, we studied the scattering by two mirrors which follow symmetrical trajectories. This example is particularly interesting since it leads to incoherent results when using the Davis-Fulling model. Instead, when using regularized amplitudes, the apparently paradoxical results are all resolved.

Appendix : Uniformly accelerated mirrors

Uniform acceleration means that (up to a 2D translation)

$$t_{cl}^2(\tau) - z_{cl}^2(\tau) = -\frac{1}{a^2} . \quad (102)$$

This equation defines two causally disconnected trajectories, lying respectively in R and L , the right and left Rindler wedges. In Sec. 2 and 3, we consider the scattering on the right of an accelerated mirror in the left quadrant. In null coordinates, its trajectory is given by

$$V_{cl}(\tau) = -\frac{1}{a}e^{-a\tau} = -1/a^2 U_{cl}(\tau) , \quad (103)$$

(we work with $dt_{cl}/d\tau > 0$).

In the self-interacting model, as shown in [17], any function $g(\tau)$ which decreases faster than $e^{-a|\tau|}$ is sufficient to obtain regular transition amplitudes. A convenient choice is provided by

$$g(\tau) \equiv e^{-2\eta \cosh(a\tau)} = e^{-\eta(aV_{cl}(\tau) + 1/aV_{cl}(\tau))} , \quad (104)$$

where $0 < \eta \ll 1$ is a dimensionless parameter. This function possesses a plateau of height 1 centered around $\tau = 0$ whose duration is given by

$$2T \simeq \frac{2|\ln(2\eta)|}{a} . \quad (105)$$

The slope of the switching on and off is independent of η and proportional to a . The tail decreases as $e^{-\eta e^{a|\tau|}}$, thus faster than the required $e^{-a|\tau|}$.

Using Eq.(104), we obtained analytical expressions for the scattering amplitudes $A_{\omega\omega'}^{ij*}$ and $B_{\omega\omega'}^{ij}$:

$$A_{\omega\omega'}^{VV*} = \delta(\omega - \omega') - \frac{4ig_0}{\pi} \frac{\sqrt{|\omega\omega'|}}{a} \frac{\eta^2}{X^2} K_2(X) , \quad (106)$$

where $X = 2\eta\sqrt{1 - i(\omega - \omega')/a\eta}$, and where K_2 is a modified Bessel function, see Appendix A in [17] and [35], and

$$A_{\omega\omega'}^{VU*} = -\frac{ig_0}{\pi} \frac{\sqrt{|\omega\omega'|}}{a} K_0(Y) , \quad (107)$$

where $Y = 2\sqrt{(\omega/a + i\eta)(-\omega'/a - i\eta)}$. The well-defined analytical properties of $A_{\omega\omega'}$ allows to obtain the pair creation amplitudes by crossing symmetry:

$$B_{\omega\omega'}^{ij} = -A_{\omega,\omega''=\omega'e^{-i\pi}}^{ij*} . \quad (108)$$

Unlike the overlaps of Eqs.(12) and (13) evaluated with the Davies-Fulling model, see Eqs.(19) of [17] for explicit expressions, these amplitudes are regular and well-defined in the complex plane of ω and ω' .

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